Joe Bottini

May 29, 2018

*Nuclear Reactor Physics*

Weston M. Stacey

Second Edition

Wiley-VCH

Foreword

This document is meant to be a derivation of various formulae associated with the  method covered in the NPRE 555 class with Professor Katy Huff in Spring 2018. The topic is covered in Stacey sections 9.6 and 9.7.

Ideas for derivation:

* Equation 9.120
  + 
  + Start with Rodrigues’ formula and the binomial theorem to prove the orthogonality relation.
* Equation 9.121
  + 
  + where 

Consider the expression for the Legendre polynomial derived in the previous derivation,



Define the operation,



This is similar to a permutation or combination operation.

Next, consider the product of two Legendre polynomials,



Combine the two sums,



Simplify,



Consider the general Legendre polynomial, . If  is even, the function  is even, and if  is odd,  is odd. Consider the pair  and the product . If  are both even or are both odd, then  is even. However, if  is an even-odd pair, then  is odd.

This is applied to the integral  to show that the integral is clearly zero if  is an even-odd pair as the integral of an odd function over a symmetric domain is zero.

Consider again Eq. (5) and apply the integral of the orthogonality condition,



Note that  is always even. Therefore, whether the exponent on  in the argument of the integral is even or odd is determined by the sum . Only when  is an even-odd pair is the sum  an odd value. In this case, the argument of the integral is an odd function, and the integral is zero for all  and  for any even-odd  pair. Therefore,



Therefore, consider the other case where  is an even quantity. If  is an even quantity, the exponent on  is even as well. The integral is therefore simplified,



For , separate the sum over ,



Move terms around,



Let’s look briefly at the operation, . The table below shows the values of  up to .

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | | | | | |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 0 | 1 |  |  |  |  |  |
| 1 | 2 |  |  |  |  |  |
| 2 | 6 | 2 |  |  |  |  |
| 3 | 20 | 12 |  |  |  |  |
| 4 | 70 | 60 | 6 |  |  |  |
| 5 | 252 | 280 | 60 |  |  |  |
| 6 | 924 | 1260 | 420 | 20 |  |  |
| 7 | 3432 | 5544 | 2520 | 280 |  |  |
| 8 | 12870 | 24024 | 13860 | 2520 | 70 |  |
| 9 | 48620 | 102960 | 72072 | 18480 | 1260 |  |
| 10 | 184756 | 437580 | 360360 | 120120 | 13860 | 252 |

Consider the operation ,



This is expressed as two combinations,



where the combination  is defined,



A couple other observations,



and



Note that the non-alternating series also yields ,

